Math 10A HW7 Solutions

(1) $$\int_0^4 \frac{1}{1+x^2}\,dx = 1\left(f(1)+f(2)+f(3)+f(4)\right)$$
$$= \left(\frac{1}{2}+\frac{1}{5}+\frac{1}{10}+\frac{1}{17}\right)$$
(let $f(x)=\frac{1}{1+x^2}$)

(2) $$\int_{-\pi}^{\pi} \sin(x)\,dx = \frac{2\pi}{4}\left(f(-\pi)+f\left(-\frac{\pi}{2}\right)+f(0)+f\left(\frac{\pi}{2}\right)\right)$$
$$= \frac{\pi}{2}\left(\sin(-\pi)+\sin\left(-\frac{\pi}{2}\right)+\sin(0)+\sin\left(\frac{\pi}{2}\right)\right)$$
(let $f(x)=\sin x$)
$$= \frac{\pi}{2}\left(0+(-1)+0+1\right)$$
$$= 0$$

(3) $$\int_0^3 e^x\,dx = 1\left(f(0.5)+f(1.5)+f(2.5)\right)$$
$$= e^{1/2}+e^{3/2}+e^{5/2}$$
(let $f(x)=e^x$)

(4) $$\int_0^3 e^x\,dx = \frac{1}{2}\left(f(0)+2f(1)+2f(2)+f(3)\right)$$
$$= \frac{1}{2}\left(e^0+2e^1+2e^2+e^3\right)$$
(let $f(x)=e^x$)
$$= \frac{1}{2}\left(1+2e+2e^2+e^3\right)$$

$E_M = \frac{K_2(b-a)^3}{24n^2}$, $E_T = \frac{K_2(b-a)^3}{12n^2}$, $E_S = \frac{K_4(b-a)^5}{180n^4}$

(5)

(a) $\int_2^3 x^2\,dx$

$f(x) = x^2$
$f'(x) = 2x$
$f''(x) = 2$

$$E_M = 5\times10^{-5} = \frac{K_2}{24N^2} = \frac{2}{24N^2}$$ midpoint rule

$$\Rightarrow N^2 = \frac{2}{24(5\times10^{-5})} \Rightarrow N = \sqrt{\frac{2}{24(5\times10^{-5})}} \approx 40.8$$

- need 41+ subintervals (midpoint)

$$E_T = 5\times10^{-5} = \frac{K_2}{12N^2} = \frac{2}{12N^2}$$ trapezoidal

$$\Rightarrow N = \sqrt{\frac{2}{12(5\times10^{-5})}} \approx 57.7$$

- need 58+ subintervals (trapezoidal)

$$E_S = 5\times10^{-4} = \frac{K_4}{180n^4} = 0$$ Simpson's rule

- need 2 subintervals (Simpson's)

(b) $\int_{-\pi}^{\pi} \sin(x)\,dx$

$f(x) = \sin(x)$ $\quad f''(x) = -\sin x$

$f'(x) = \cos x$ $\quad f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$$E_M = 5 \times 10^{-5} = \frac{K_2 (b-a)^3}{24n^2} = \frac{(2\pi)^3}{24N^2}$$

$$\Rightarrow N = \sqrt{\frac{(2\pi)^3}{24 \cdot 5 \cdot 10^{-5}}} \approx 454.65$$

- need 455+ subintervals (midpoint)

$$E_T = 5 \times 10^{-5} = \frac{K_2 (2\pi)^3}{12N^2}$$

$$\Rightarrow N = \sqrt{\frac{(2\pi)^3}{12 \cdot 5 \cdot 10^{-5}}} \approx 642.97$$

- need 643+ subintervals (trapezoidal)

$$E_S = 5 \times 10^{-5} = \frac{K_4 (2\pi)^5}{180N^4} = \frac{(2\pi)^5}{180N^4}$$

$$\Rightarrow N = \sqrt[4]{\frac{(2\pi)^5}{180 \cdot 5 \cdot 10^{-5}}} \approx 32.3$$

- need 33+ subintervals (Simpson's)

(c) $\int_1^5 \frac{1}{x}\,dx$

$f(x) = x^{-1}$

$f'(x) = -x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f^{(4)}(x) = 24x^{-5}$

$$E_M = 5\cdot 10^{-5} = \frac{K_2 (4)^3}{24N^2} = \frac{2(4)^3}{24N^2}$$

$$\Rightarrow N = \sqrt{\frac{64}{12\cdot 5\cdot 10^{-5}}} \approx 326.6$$

· need 327+ subintervals (midpoint)

$$E_T = 5\cdot 10^{-5} = \frac{K_2 (4)^3}{12N^2} = \frac{2\cdot 64}{12\cdot N^2}$$

$$N = \sqrt{\frac{64}{6\cdot 5\cdot 10^{-5}}} \approx 461.9$$

· need 462+ subintervals (trapezoidal)

$$E_S = 5\cdot 10^{-5} = \frac{K_4 (4)^5}{180N^4} = \frac{24\cdot 1024}{180N^4}$$

$$\Rightarrow N = \sqrt[4]{\frac{24\cdot 1024}{180\cdot 5\cdot 10^{-5}}} \approx 40.6$$

· need 41+ subintervals (Simpson's)

(6) $\int_0^{\pi} \sin(x)\,dx$

$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$

$E_M = 0.5 = \frac{K_2 \pi^3}{24N^2} = \frac{\pi^3}{24N^2}$

$\Rightarrow N = \sqrt{\frac{\pi^3}{24 \cdot 1/2}} \approx 1.6$ (need 2 subintervals)

So $\int_0^{\pi} \sin(x)\,dx = \frac{\pi}{2}\left(f\left(\frac{\pi}{4}\right) + f\left(\frac{3\pi}{4}\right)\right)$
$= \pi/2\,(1/\sqrt{2} + 1/\sqrt{2})$
$= \pi/2\,(\sqrt{2})$

(7) $\int_{-1}^{1} e^{-x^2}\,dx$ $\quad \Delta x = 2/4 = 1/2$

$= \frac{1}{6}\left(f(-1) + 4f\left(-\frac{1}{2}\right) + 2f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right)$

$= \frac{1}{6}\left(e^{-1} + 4e^{-1/4} + 2 + 4e^{-1/4} + e^{-1}\right)$

(8) True!

(9) True! (We have $|E_T| \leq \frac{K_2 (b-a)^3}{12N^2} = 0$)

(10) True! (We have $|E_s| \leq \frac{K_4 (b-a)^5}{180N^4}$

but notice if $f(x) = x^3 - 3x$ then $f^{(4)}(x) = 0$

so $|E_s| \leq 0$)